CLERK MAXWELL'S CORRECTIONS TO THE PAGE PROOFS OF "A DYNAMICAL THEORY OF THE ELECTROMAGNETIC FIELD".

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Introduction.

The Johns Hopkins University Library is fortunate in possessing the corrected page proofs of Maxwell's paper "A Dynamical Theory of the Electromagnetic Field"[1]. They form part of a collection of proofs which was originally in the possession of Sir George Stokes, into whose hands they doubtless came when he was Secretary of the Royal Society. Since it was in this paper, read before the Royal Society in December 1864, that Maxwell presented his electromagnetic theory in its most developed form, it is of interest to examine the corrections he saw fit to make in the page proofs and to discuss their significance.

The proofs appear to have been corrected by Maxwell at his country home during the summer of 1865: they bear a printer's note "Aug. 10/65", and a note by Maxwell "To Printer. Address—Glenlair, Dalbeattie N.B.". The alterations, which are in Maxwell's hand, include ordinary corrections of typographical errors, minor changes in style, and an admonition to the printer to "change heads throughout" from "J. C. Maxwell" to "Clerk Maxwell". Changes in scientific content occur in a discussion of the value of the divergence of the vector potential and in a discussion of the optical properties of gold leaf.

Notation.

The following symbols are used:

$\mathbf{A}$: vector potential.
J: div $\mathbf{A}$.
χ: a scalar such that div **grad** $\chi=J$.
F, G, H: components of $\mathbf{A}$.
F', G', H': components of $(\mathbf{A}-\mathbf{grad}\ \chi)$.
e: "free positive electricity per unit volume".
V: wave velocity.
k: "ratio of the electromotive force to the electric displacement".

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[1] *Phil. Trans.*, 1865, **155**, 459–512; *Scientific Papers*, Cambridge, 1890, vol. I, pp. 526–597.

In the discussion which follows, Maxwell's Cartesian notation is replaced by vector notation. The symbol **A** refers to the vector identified by Maxwell in his paper by its components, F, G, H. This is the same quantity used by Maxwell in his treatise and there variously referred to as " the vector potential of magnetic induction ", " the electrokinetic momentum " and " the electromagnetic momentum " [2]. The notation ($\mathbf{A}-\mathbf{grad}\ \chi$) refers to the vector of components

$$F'=F-\frac{d\chi}{dx},\ G'=G-\frac{d\chi}{dy},\ H'=H-\frac{d\chi}{dz}.$$

MAXWELL'S CORRECTIONS.

If a vector **A** satisfies the relation $\mathbf{B}=\mathbf{curl}\ \mathbf{A}$ then the vector ($\mathbf{A}+\mathbf{grad}$ scalar) satisfies the same relation. Hence the vector potential **A** is indeterminate and is not fully specified until its divergence is given. The common practice in electromagnetic theory is to assign to div **A** a value appropriate to and useful in the problem under consideration.

This is the modern view. Maxwell, however, proceeds otherwise. Using the properties of the vector potential **A**, and employing a derivation in which $J(=\text{div}\ \mathbf{A})$ drops out of the equations, he shows the existence of propagated magnetic waves [3]. He later examines the possibility that variations in J may contribute to the propagated waves [4]. He defines a scalar χ such that div $\mathbf{grad}\ \chi=J$, and then defines a new vector whose components are $F-\frac{d\chi}{dx}$, $G-\frac{d\chi}{dy}$, and $H-\frac{d\chi}{dz}$, that is, a vector ($\mathbf{A}-\mathbf{grad}\ \chi$). Obviously div ($\mathbf{A}-\mathbf{grad}\ \chi$) is zero no matter what J may be. Using the fact that div ($\mathbf{A}-\mathbf{grad}\ \chi$) is zero he shows that variation in ($\mathbf{A}-\mathbf{grad}\ \chi$) will give rise to propagated disturbances [5].

He now proceeds to investigate the possible effect of the introduction of χ, i.e. the effect of the value of J, and finds J is subject only to the condition

$$ke=\frac{dJ}{dt}-k\nabla^2\phi(x, y, z)$$

and says, in the original proof :

> " Hence J is either zero or it continually increases or diminishes with the time, if e remains constant, which no physical quantity can do. Hence J is zero, and the only disturbance propagated is that indicated by F', G', H', which is wholly transversal."

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[2] *A Treatise on Electricity and Magnetism,* 3rd ed., London, 1892 and 1946, Vol. II.

[3] *Phil. Trans.*, 1865, **155**, 497–499 ; *Papers*, I, pp. 577–579.

[4] *Ibid.*, p. 499 ; *Papers*, I, p. 580.

[5] *Ibid.*, p. 500 ; *Papers*, I, p. 581.

The above passage is replaced in the published paper by:

> "Since the medium is a perfect insulator, e, the free electricity, is immoveable, and therefore dJ/dt is a function of x, y, z, and the value of J is either constant or zero, or uniformly increasing or diminishing with the time; so that no disturbance depending on J can be propagated as a wave"[6].

From this it appears that Maxwell began using **A** in the derivation of the wave equation, found that the value of div **A** might be important, and therefore introduced a vector the divergence of which would be zero no matter what div **A** might be. Having repeated the demonstration in terms of the new vector, he then presented a physical argument to show that div $\mathbf{A}=0$. If this argument had been retained he need not have introduced χ at all. In altering his proofs he returned to the position implicit in the introduction of χ: that the major result (the existence of transversal waves which depend for their origin upon variation of **A**) may be obtained without specifying the value of div **A**. In a sense Maxwell overlooked the obvious formal point that the value of div **A** may be assigned at convenience, while nevertheless establishing the fundamental physical point.

The other change in scientific content occurs in a discussion of experimental results. Maxwell's theory yields a quantitative relationship between the optical and electrical properties of matter, and this relationship fails badly in predicting the optical properties of certain metals[7]. On the subject of the optical properties of gold leaf the original proof says:

> "If the resistance of gold is the same for electromotive forces of short period as for those with which we experimented, the amount of light which passes through a thickness of 1/282000 inch would be only 10^{-300} of the incident light, a totally imperceptible quantity. I find that between 1/500 and 1/900 of green light gets through such gold leaf."

Whereas the paper as printed says:

> "If the resistance of gold is the same for electromotive forces of short period as for those with which we make experiments, the amount of light which passes through a piece of gold-leaf of which the resistance was determined by Mr. C. Hockin, would be only 10^{-50} of the incident light, a totally imperceptible quantity. I find that between 1/500 and 1/1000 of green light gets through such gold-leaf. Much of this is transmitted through holes and cracks; there is enough, however, transmitted through the gold itself to give a strong green hue to the transmitted light"[8].

[6] *Ibid.*, p. 500, ll. 23–26; *Papers*, I, p. 582, ll. 3–6.

[7] Sir Edmund T. Whittaker, *History of the Theories of Aether and Electricity*, New York, 1951, vol. I, p. 164.

[8] *Phil. Trans.*, *loc. cit.*, p. 504, *Papers*, I, p. 587.

It appears that the same piece of gold leaf is under discussion, because of the recurrence of the approximate range 1/500 to 1/1000. The calculated transmission has changed in the direction of agreement with measurement by a factor of 10^{250}, a change that may have resulted from the use of the measured resistance in place of one calculated from the thickness. Maxwell was at any rate actively concerned with this prediction which departs so widely from measurement.

There is a note by the printer which suggests that another important calculation was left for rechecking : alongside the value for the calculated " velocity of light in the space surrounding the earth " is the note " Pencil writing in MS "[9].

We close with a correction which reveals Maxwell's personal feelings about the significance of the propagated electromagnetic waves. In the section on " Disturbances in a Crystallized Medium " the published paper in one place reads :

> "The solution $V=0$ refers to a case in which there is no propagation "[10].

Maxwell had originally written rather more bluntly :

> "The solution $V=0$ is unimportant."

This paper was prepared at the Thomas C. Jenkins Laboratories of Biophysics of The Johns Hopkins University during the tenure of a National Research Council Rockefeller Foundation Post-Doctoral Fellowship in the Medical Sciences.

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[9] *Ibid.*, p. 499, l. 20; *Papers*, I, p. 580, l. 11.

[10] *Ibid.*, p. 502, l. 30; *Papers*, I, p. 584, l. 21.